Chapter - Motion in a Plane

Topic-1: Vectors

1 MCQs with One Correct Answer

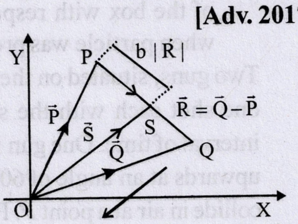
1. Three vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among vectors $\vec{P}, \vec{Q}$ and $\vec{S}$ is **[Adv. 2017]**

(a) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$

(b) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$

(c) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

(d) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$



3 Numeric / New Stem Based Questions

2. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a(\cos \omega t\, \hat{i} + \sin \omega t\, \hat{j})$, where a is a constant and $\omega = \pi/6$ rad s^{-1}. If $|\vec{A}+\vec{B}| = \sqrt{3}\,|\vec{A}-\vec{B}|$ at time $t = \tau$ for the first time, the value of τ, in *seconds*, is ________. **[Adv. 2018]**

Topic-2: Motion in a Plane with Constant Acceleration

2 Integer Value Answer

1. A ball is thrown from the location $(x_0, y_0) = (0,0)$ of a horizontal playground with an initial speed v_0 at an angle θ_0 from the $+x$-direction. The ball is to be hit by a stone, which is thrown at the same time from the location $(x_1, y_1) = (l, 0)$. The stone is thrown at an angle $(180 - \theta_1)$ from the $+x$-direction with a suitable initial speed. For a fixed v_0, when $(\theta_0, \theta_1) = (45°, 45°)$, the stone hits the ball after time T_1, and when $(\theta_0, \theta_1) = (60°, 30°)$, it hits the ball after time T_2. In such a case, $\left(\frac{T_1}{T_2}\right)^2$ is ______. **[Adv. 2024]**

6 MCQs with One or More than One Correct Answer

2. Starting at time $t = 0$ from the origin with speed 1 ms^{-1}, a particle follows a two-dimensional trajectory in the x-y plane so that its coordinates are related by the equation $y = \frac{x^2}{2}$. The x and y components of its acceleration are denoted by a_x and a_y, respectively. Then **[Adv 2020]**

(a) $a_x = 1$ ms^{-2} implies that when the particle is at the origin, $a_y = 1$ ms^{-2}

(b) $a_x = 0$ implies $a_y = 1$ ms^{-2} at all times

(c) at $t = 0$, the particle's velocity points in the x-direction

(d) $a_x = 0$ implies that at $t = 1$ s, the angle between the particle's velocity and the x axis is 45°

3. The coordinates of a particle moving in a plane are given by $x(t) = a\cos(pt)$ and $y(t) = b\sin(pt)$ where $a, b\,(< a)$ and p are positive constants of appropriate dimensions. Then **[1999S - 3 Marks]**

(a) the path of the particle is an ellipse

(b) the velocity and acceleration of the particle are normal to each other at $t = \pi/(2p)$

(c) the acceleration of the particle is always directed towards a focus

(d) the distance travelled by the particle in time interval $t = 0$ to $t = \pi/(2p)$ is a

Topic-3: Projectile Motion

1 MCQs with One Correct Answer

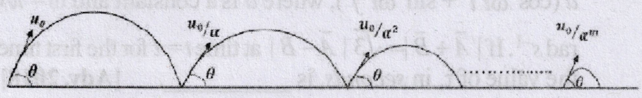
1. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10$ m/s^2, the equation of its trajectory is : **[2012]**

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$

(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

2 Integer Value Answer

2. A ball is thrown from ground at angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is $0.8\ V_1$, the value of α is ____ **[Adv. 2019]**



3. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is **[Adv. 2011]**

3 Numeric Answer/New Stem Based Questions

4. A projectile is fired from horizontal ground with speed v and projection angle θ. When the acceleration due to gravity is g, the range of the projectile is d. If at the highest point in its trajectory, the projectile enters a different region where the effective acceleration due to gravity is $g' = \dfrac{g}{0.81}$, then the new range is $d' = nd$. The value of n is _____. **[Adv. 2022]**

A projectile is thrown from a point O on the ground at an angle 45° from the vertical and with a speed $5\sqrt{2}$ m/s. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down to the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x meters from the point O. The acceleration due to gravity $g = 10$ m/s^2.

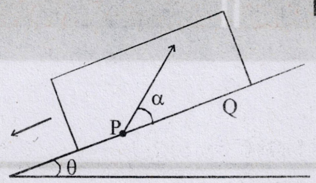
5. The value of t is _____. **[Adv. 2021]**
6. The value of x is _____. **[Adv. 2021]**

5 True / False

7. A projectile fired from the ground follows a parabolic path. The speed of the projectile is minimum at the top of its path. **[1984 - 2 Marks]**

10 Subjective Problems

8. A large, heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to the box is u, and the direction of projection makes an angle α with the bottom as shown in Figure. **[1998 - 8 Marks]**

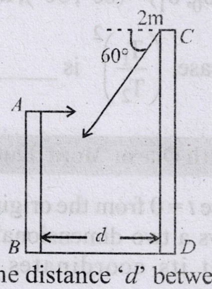


(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when particle was projected.

9. Two guns, situated on the top of a hill of height 10 m, fire one shot each with the same speed $5\sqrt{3}$ m s^{-1} at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at a point P. Find (i) the time-interval between the firings, and (ii) the coordinates of the point P. Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane. **[1996 - 5 Marks]**

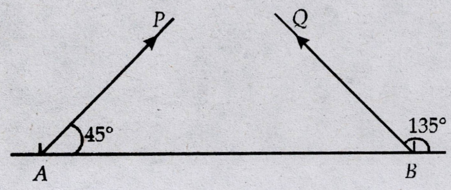
10. Two towers AB and CD are situated a distance d apart as shown in figure.
AB is 20 m high and CD is 30 m high from the ground. An object of mass m is thrown from the top of AB horizontally with a velocity of 10 m/s towards CD. **[1994 - 6 Marks]**
Simultaneously another object of mass 2 m is thrown from the top of CD at an angle of 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects move in the same vertical plane, collide in mid-air and stick to each other.



(i) Calculate the distance 'd' between the towers and,

(ii) Find the position where the objects hit the ground.

11. Particles P and Q of mass 20 gm and 40 gm respectively are simultaneously projected from points A and B on the ground. The initial velocities of P and Q make 45° and 135° angles respectively with the horizontal AB as shown in the figure. Each particle has an initial speed of 49 m/s. The separation AB is 245 m. **[1982 - 8 Marks]**



Both particle travel in the same vertical plane and undergo a collision After the collision, P retraces its path, Determine the position of Q when it hits the ground. How much time after the collision does the particle Q take to reach the ground? Take $g = 9.8$ m/s^2.

Topic-4: Relative Velocity in Two Dimensions & Uniform Circular Motion

1 MCQs with One Correct Answer

1. A boat which has a speed of 5 km/hr in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/hr is **[1988 - 1 Mark]**

(a) 1 (b) 3 (c) 4 (d) $\sqrt{41}$

2. A river is flowing from west to east at a speed of 5 metres per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still water, wants to swim across the river in the shortest time. He should swim in a direction **[1983 - 1 Mark]**

(a) due north (b) 30° east of north
(c) 30° west of north (d) 60° east of north

Answer Key

Topic-1 : Vectors

1. (a) 2. (2.00)

Topic-2 : Motion in a Plane with Constant Acceleration

1. (2) 2. (a, b, c, d) 3. (a, b, c)

Topic-3 : Projectile Motion

1. (b) 2. (4) 3. (5) 4. (0.95) 5. (0.50) 6. (7.50) 7. (True)

Topic-4 : Relative Velocity in two Dimensions & Uniform Circular Motion

1. (b) 2. (a)

Hints & Solutions

Topic-1: Vectors

1. **(a)** Here $\vec{P}+b\vec{R}=\vec{S} \Rightarrow \vec{R}=\dfrac{\vec{S}-\vec{P}}{b}$

Also $\vec{R}=\vec{Q}-\vec{P}$

$\therefore \quad \dfrac{\vec{S}-\vec{P}}{b}=\vec{Q}-\vec{P} \Rightarrow \vec{S}-\vec{P}=b\vec{Q}-b\vec{P}$

$\therefore \quad \vec{S}=b\vec{Q}+(1-b)\vec{P}$

2. **(2.00)** Given : $|\vec{A}+\vec{B}|=\sqrt{3}|\vec{A}-\vec{B}|$

$\therefore \quad |a\,\hat{i}+a\cos\omega t\,\hat{i}+a\sin\omega t\,\hat{j}|=\sqrt{3}|a\,\hat{i}-a\cos\omega t\,\hat{i}-a\sin\omega t\,\hat{j}|$

$\Rightarrow \quad |(1+\cos\omega t)\hat{i}+\sin\omega t\,\hat{j}|=\sqrt{3}|(1-\cos\omega t)\hat{i}-\sin\omega t\,\hat{j}|$

$\sqrt{2+2\cos\omega t}=\sqrt{3}\,\sqrt{2-2\cos\omega t}$

$\therefore \quad 1+\cos\omega t=3(1-\cos\omega t)$

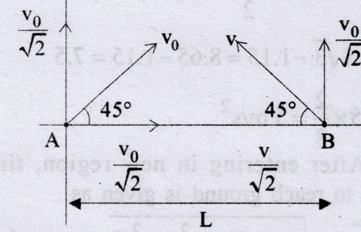
$\Rightarrow \quad 4\cos\omega t=2 \quad \therefore \cos\omega t=\dfrac{1}{2} \quad$ or, $\omega t=\dfrac{\pi}{3}$

$\therefore \quad \dfrac{\pi}{6}\times\tau=\dfrac{\pi}{3} \quad \therefore \tau=2.00$ seconds

Topic-2: Motion in a Plane with Constant Acceleration

1. **(2)** **For case I :** For collision $\dfrac{v_0}{\sqrt{2}}=\dfrac{v}{\sqrt{2}}$

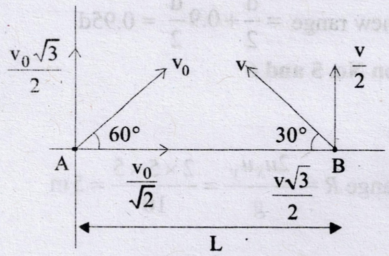
$\therefore \quad v=v_0$



So $\quad T_1=\dfrac{L}{\dfrac{v_0}{\sqrt{2}}+\dfrac{v}{\sqrt{2}}}=\dfrac{L}{\sqrt{2}v_0}$

For case II : For collision, $\dfrac{v_0\sqrt{3}}{2}=\dfrac{v}{2}$

$\therefore \quad v=\sqrt{3}v_0$

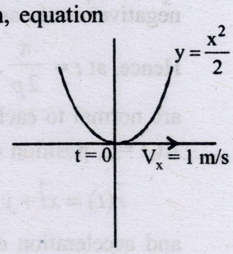


So, $\quad T_2=\dfrac{L}{\dfrac{v_0}{2}+v\dfrac{\sqrt{3}}{2}}T_2=\dfrac{L}{\dfrac{v_0}{2}+\dfrac{3v_0}{2}}=\dfrac{L}{2v_0}$

So, $\left(\dfrac{T_1}{T_2}\right)^2\left(\dfrac{L/\sqrt{2}v_0}{L/2v_0}\right)^2=(\sqrt{2})^2=2$

2. **(a, b, c, d)** According to question, equation

$y=\dfrac{x^2}{2}$



At t = 0, $\left.\begin{matrix}x=0, y=0\\ u=1\end{matrix}\right\}$ given

$y=\dfrac{x^2}{2}$

$\dfrac{dy}{dt}=\dfrac{1}{2}\cdot 2x\dfrac{dx}{dt}=x\dfrac{dx}{dt} \Rightarrow v_y=xv_x$

Now differenciate wrt time

$\dfrac{d^2y}{dt^2}=x\dfrac{d^2x}{dt^2}+\left(\dfrac{dx}{dt}\right)^2$

$a_y=\dfrac{dx}{dt}\cdot V_x+xa_x$

$a_y=v_x^2+xa_x$

(a) If $a_x=1$ and particle is at origin (x = 0, y = 0)

$a_y=v_x^2$

$a_y=1^2=1$ m/s^2

(b) $a_x=0$

$a_y=v_x^2+xa_x \Rightarrow a_y=v_x^2$

If $a_x=0$, v_x = constant = 1 $\Rightarrow a_y=1^2=1$

(c) At t = 0, x = 0 $\quad v_y=xv_x$

speed =1; $\quad v_y=0 \Rightarrow v_x=1$

(d) $a_x=0$ implies that at t = 1s

$a_y=v_x^2+xa_x \Rightarrow v_y=xv_x \Rightarrow a_y=v_x^2$

If $a_x=0 \Rightarrow V_x$ = constant initially ($v_x=1$) $\Rightarrow a_y=1^2=1$

At t = 1 sec

$v_y=0+a_y\times t=1\times 1=1$

$\tan\theta=\dfrac{V_y}{V_x}=x \quad (\theta \rightarrow$ angle with x axis)

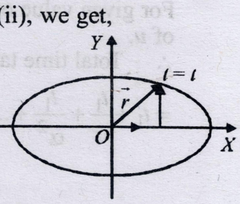
$\therefore \tan\theta=\dfrac{V_y}{V_x}=\dfrac{1}{1}=1 \Rightarrow \theta=45°$

3. **(a, b, c)** $x=a\cos pt \Rightarrow \cos(pt)=\dfrac{x}{a}$ ... (i)

$y=b\sin pt \Rightarrow \sin(pt)=\dfrac{y}{b}$ ... (ii)

Squaring and adding eqn. (i) and (ii), we get,

$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$



Hence path of the particle is an ellipse.

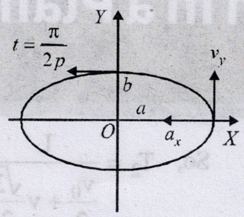
From the given equations

we can find,

$$\frac{dx}{dt} = v_x = -ap \sin pt; \quad \frac{d^2x}{dt^2} = a_x = -ap^2 \cos pt$$

$$\frac{dy}{dt} = v_y = \text{pb} \cos pt$$

and $$\frac{d^2y}{dt^2} = a_y = -bp^2 \sin pt$$



At time $t = \frac{\pi}{2p}$ or $pt = \frac{\pi}{2}$

a_x and v_y become zero (because $\cos\frac{\pi}{2} = 0$). Only v_x and a_y are left, or we can say that velocity is along negative x-axis and acceleration along negative y-axis.

Hence, at $t = \frac{\pi}{2p}$, velocity and acceleration of the particle are normal to each other.

At $t = t$, position of the particle

$$\vec{r}(t) = x\hat{i} + y\hat{j} = a\cos pt\,\hat{i} + b\sin pt\,\hat{j}$$

and acceleration of the particle

$$\vec{a}(t) = a_x\hat{i} + a_y\hat{j} = -p^2[a\cos pt\,\hat{i} + b\sin pt\,\hat{j}]$$

$$= -p^2[x\hat{i} + y\hat{j}] = -p^2\vec{r}(t)$$

Therefore, acceleration of the particle is always directed towards origin.

At $t = 0$, particle is at $(a, 0)$ and at $t = \frac{\pi}{2p}$, particle is at $(0, b)$. Therefore, the distance covered is one fourth of the elliptical path and not a.

Topic-3: Projectile Motion

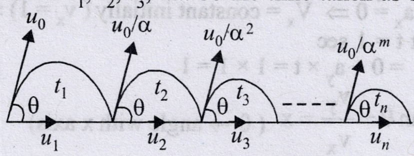
1. **(b)** From equation, $\vec{v} = \hat{i} + 2\hat{j}$

$\Rightarrow x = t$ ...(i)

$y = 2t - \frac{1}{2}(10t^2)$ ...(ii)

From (i) and (ii), $y = 2x - 5x^2$

2. **(4)** Let $u_1, u_2, u_3, \ldots$ be the horizontal velocity of the projectiles and $t_1, t_2, t_3, \ldots$ be the time taken as shown in figure.



Average velocity $= \frac{\text{total displacement}}{\text{total time taken}}$

Time of flight $T = \frac{2u\sin\theta}{g}$

For given value of θ, value of T will change with the value of u.

$\therefore$ Total time taken $= t_1 + t_2 + t_3 + \ldots$

$$= t_1 + \frac{t_1}{\alpha} + \frac{t_1}{\alpha^2} + \ldots = \frac{t_1}{1-\frac{1}{\alpha}} = \frac{t_1\alpha}{(\alpha-1)}$$

Total displacement $= u_1t_1 + u_2t_2 + u_3t_3 + \ldots$

$$= u_1t_1 + \frac{u_1}{\alpha}\cdot\frac{t_1}{\alpha} + \frac{u_1}{\alpha^2}\cdot\frac{t_1}{\alpha^2} + \ldots = \frac{u_1t_1}{1-\frac{1}{\alpha^2}} = \frac{u_1t_1\alpha^2}{(\alpha^2-1)}$$

$\therefore$ Average velocity $= \frac{\text{total displacement}}{\text{total time}}$

$$= \frac{u_1t_1\alpha^2}{(\alpha^2-1)} \times \frac{(\alpha-1)}{t_1\alpha} = \frac{u_1\alpha}{\alpha+1}$$

According to question, average velocity $= 0.8\,V_1$

or $0.8\,V_1 = \frac{u_1\alpha}{\alpha+1}$ ..... (i)

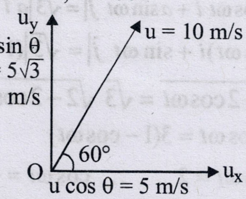
$V_1 = \frac{u_1 \times t_1}{t_1} = u_1$ ..... (ii)

From equation (i) and (ii)

$\alpha = 0.8\,\alpha + 0.8$ or $\alpha - 0.8\,\alpha = 0.8$

or $0.2\,\alpha = 0.8$ or $\alpha = \frac{0.8}{0.2} = 4$

3. **(5)** Here, $u = 10$ m/s, $\theta = 60°$



$\therefore\ u_x = u\cos\theta = 10 \times \cos 60° = 5$ m/s

and $u_y = u\sin\theta = 10 \times \sin 60° = 5\sqrt{3}$ m/s

$\therefore\ t = \frac{2u_y}{g} = \frac{2\times5\sqrt{3}}{10} = \sqrt{3}$ s

Let a be the acceleration of train.

$$1.15 = u_xt - \frac{1}{2}at^2$$

$$\Rightarrow 1.15 = 5\times\sqrt{3} - \frac{1}{2}\times a\times(\sqrt{3})^2$$

$$\Rightarrow \frac{3a}{2} = 5\sqrt{3} - 1.15 = 8.65 - 1.15 = 7.5$$

$$\therefore\ a = 7.5\times\frac{2}{3} = 5\text{ m/s}^2$$

4. **(0.95)** After entering in new region, time taken by projectile to reach ground is given as

$$t = \sqrt{\frac{2h}{g_{eff}}} = \sqrt{\frac{2\times0.81\times u^2\sin^2\theta}{g\times2g}} = 0.9\frac{u\sin\theta}{g}$$

So, horizontal displacement done by projectile in new region is given as

$$x = 0.9\times\frac{u\sin\theta}{g}\times u\cos\theta = 0.9\left(\frac{d}{2}\right)$$

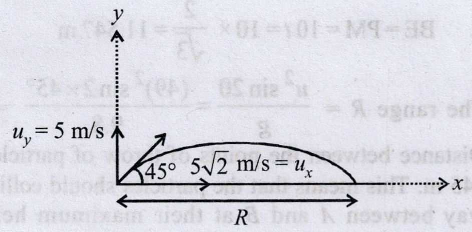
Now, new range $= \frac{d}{2} + 0.9\frac{d}{2} = 0.95d$

For Question No. 5 and 6

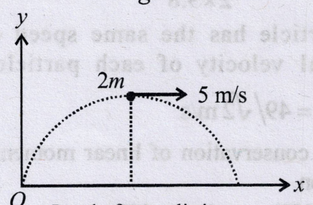
5. **(0.50)**

6. **(7.50)**

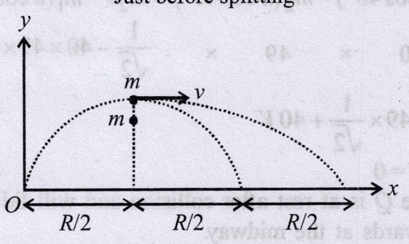
Sol. Range $R = \frac{2u_xu_y}{g} = \frac{2\times5\times5}{10} = 5\,\text{m}$



Time of flight $T = \dfrac{2u_y}{g} = \dfrac{2 \times 5}{10} = 1$ second



Just before splitting



Just after splitting

$\because$ Time of motion of one part falling vertically downwards = 0.5 sec = $\dfrac{T}{2}$

From law conservation of momentum, $P_i = P_f$

$2m \times 5 = m \times v \Rightarrow v = 10$ m/s

As there is no initial velocity in vertical direction for second particle, so it will also take same time as first one.

$\therefore$ Time of motion of another part, $t = \dfrac{T}{2}$ = **0.5 sec**

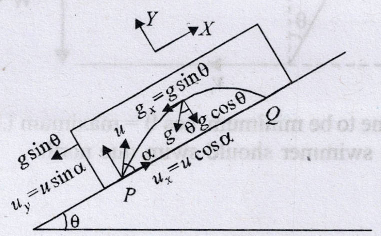
Displacement of other part in 0.5 sec in horizontal direction = $v\dfrac{T}{2} = 10 \times 0.5 = 5\text{ m} = R$

$\therefore$ Total distance of second part from point 'O' is,

$$x = \frac{3R}{2} = 3 \times \frac{5}{2} \Rightarrow x = \mathbf{7.5\ m}$$

7. **(True)** T.E. = P.E. + K.E. = Constant
At the top, K.E. is minimum and P.E. is maximum. Since K.E. is minimum so speed is also minimum.

8. **(a)** The relative velocity of the particle with respect to the box is u. Now the relative velocity in x and y-direction u_x and u_y respectively.
Since, there is no velocity of the box in the y-direction, therefore this is the vertical velocity of the particle with respect to ground also.



Taking relative terms w.r.t. box Y-direction motion

$u_y = +u \sin\alpha$

$a_y = -g\cos\theta$

$s_y = 0$ (activity is taken till the time the particle comes back to the box.)

$t_y = t$

$$s_y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow 0 = (u\sin\alpha)\,t - \frac{1}{2}g\cos\theta \times t^2$$

$$\Rightarrow \quad t = 0 \text{ or } t = \frac{2u\sin\alpha}{g\cos\theta}$$

Taking relative terms w.r.t. box X - direction motion

$u_x = +u\cos\alpha$; $a_x = 0$, $t_x = t$, $s_x = s_x$

$$s_x = u_x t + \frac{1}{2}a_x t^2 \Rightarrow s_x = u\cos\alpha \times \frac{2u\sin\alpha}{g\cos\theta} = \frac{u^2\sin 2\alpha}{g\cos\theta}$$

(b) The distance travelled by the box in time $\left(\dfrac{2u\sin\alpha}{g\cos\theta}\right)$ should be equal to the range of the particle for the observer on ground to see horizontal displacement to be zero. Let the speed of the box at the time of projection of particle be U. Then for the motion of box with respect to ground.

$u_x = -U$; $a_x = -g\sin\theta$;

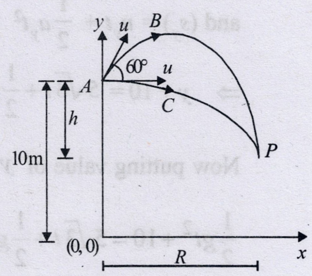
$$t_y = \frac{2u\sin\alpha}{g\cos\theta};\ s_x = \frac{-u^2\sin 2\alpha}{g\cos\theta}$$

$$s_x = u_x t + \frac{1}{2}a_x t^2$$

$$\frac{-u^2\sin 2\alpha}{g\cos\theta} = -U\left(\frac{2u\sin\alpha}{g\cos\theta}\right) - \frac{1}{2}g\sin\theta\left(\frac{2u\sin\alpha}{g\cos\theta}\right)^2$$

Solving eqn. (i) we get $U = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$

9. Till the bullets collide in air at point P they follow different path ACP and ABP.
Let t_1 = Time taken by first bullet in reaching P.
t_2 = Time taken by second bullet in reaching P.



For path ABP
Range, $R = (u\cos 60°)t_2$
Range, $R = (ut_1)$
For path ACP

$$\therefore \quad \frac{ut_2}{2} = ut_1$$

or $\quad t_2 = 2t_1 \quad$ .... (i)

Height for path ABP

$$h = -(u\sin 60°)t_2 + \frac{1}{2}gt_2^2$$

$$h = \frac{1}{2}gt_1^2$$

Height for path ACP $\quad \therefore \quad \dfrac{-u \times \sqrt{3}t_2}{2} + \dfrac{1}{2}gt_2^2 = \dfrac{1}{2}gt_1^2$

or $\quad \dfrac{-5\sqrt{3} \times \sqrt{3}t_2}{2} + \dfrac{10t_2^2}{2} = \dfrac{10t_1^2}{2}$

or $\quad -15t_2 + 10t_2^2 = 10t_1^2$ or $-3t_2 + 2t_2^2 = 2t_1^2$

or $\quad -(3 \times 2t_1) + 2(2t_1)^2 = 2t_1^2$

or $\quad -6t_1 + (4 \times 2)t_1^2 = 2t_1^2$ or $6t_1 = 6t_1^2$

or $\quad t_1 = 1$ sec $\quad$ .... (ii)

$\therefore \quad t_2 = 2t_1 = 2$ sec .... (iii)

Now $R = ut_1$ for path ACP

or $\quad R = 5\sqrt{3}\times 1 = 5\sqrt{3}$ m .... (iv)

Again $h = \frac{1}{2}gt_1^2$ for path ACP

$= \frac{1}{2}\times 10\times(1)^2 = 5$ m

$\therefore$ y-coordinate of $P = 10 - h = 10 - 5 = 5$ m .... (v)

(i) Time interval between the firings $= (t_2 - t_1)$
$= (2 - 1) = 1$ second

(ii) Coordinates of point, $P = (R, y) = (5\sqrt{3}$ m, 5 m$)$

10. (i) Let t be the time taken for collision.
Till their point of collision objects thrown from A and C run along parabolic paths.

For mass m thrown horizontally from A.

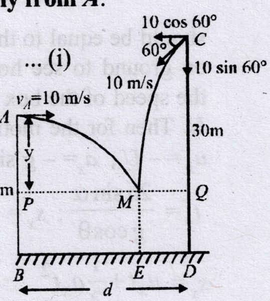
For horizontal motion

$PM = V_A \times t = 10\,t$ ... (i)

For vertical motion

$u_y = 0;\ s_y = y;\ a_y = g;\ t_y = t$

$\therefore \quad y = \frac{1}{2}gt^2$ ... (ii)



Velocity in vertical direction

$v_y = u_y + a_y t = gt$ ... (iii)

For mass 2m thrown from C

For horizontal motion $QM = [10 \cos 60°]\ t$

$QM = 5\,t$ ... (iv)

For vertical motion $v_y = 10 \sin 60° = 5\sqrt{3}$; $a_y = g$

$s_y = y + 10;\ t_y = t$

Now, $v_y = 5\sqrt{3} + gt$ ... (v)

and $(s_y) = u_y t + \frac{1}{2}a_y t^2$

$\Rightarrow \quad y + 10 = 5\sqrt{3}\,t + \frac{1}{2}gt^2$ ... (vi)

Now putting value of $y = \frac{1}{2}gt^2$ in eqn. (vi)

$\frac{1}{2}gt^2 + 10 = 5\sqrt{3}\,t + \frac{1}{2}gt^2 \Rightarrow t = \frac{2}{\sqrt{3}}$ sec

Distance between to towers

$BD = PM + MQ = 10\,t + 5\,t = 15\,t = 15\times\frac{2}{\sqrt{3}} = 17.32$ m

(ii) Applying conservation of linear momentum (during collision of the masses at M) in the horizontal direction

Let velocity of combined mass along horizontal $= v_x$

$m \times 10 - 2\,m\ 10 \cos 60° = 3\,m \times v_x$

$\Rightarrow \quad 10\,m - 10\,m = 3\,m \times v_x \Rightarrow v_x = 0$

Since, the horizontal momentum comes out to be zero, the combination of masses will drop vertically downwards and fall at E.

$\therefore \quad BE = PM = 10\,t = 10\times\frac{2}{\sqrt{3}} = 11.547$ m

11. The range $R = \frac{u^2 \sin 2\theta}{g} = \frac{(49)^2 \sin 2\times 45°}{9.8} = 245$ m.

Distance between the points of throw of particles is also 245 m. This means that the particles should collide at mid way between A and B at their maximum height $H = \frac{u^2\sin^2\theta}{2g} = \frac{(49)^2(\sin 45°)^2}{2\times 9.8} = 61.25$ m

Each particle has the same speed of 49 m/s and horizontal velocity of each particle is same i.e.,

$u\cos 45° = 49/\sqrt{2}$ m/s

Applying conservation of linear momentum at the point of collision

$m_1(u\cos 45°) - m_2(u\cos 45°) = m_2 v - m_1(u\cos 45°)$

$20 \times 49 \times \frac{1}{\sqrt{2}} - 40\times 49\times\frac{1}{\sqrt{2}} = -20\times 49\times\frac{1}{\sqrt{2}} + 40\,V$

$\therefore \quad V = 0$

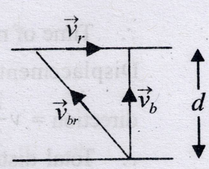
Particle Q is at rest after collision and will fall vertically downwards at the midway.

Time taken $= \sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times 61.25}{9.8}} = 3.53$ s

Topic-4: Relative Velocity in Two Dimensions & Uniform Circular Motion

1. **(b)** Shortest route corresponds to $\vec{v}_b$ perpendicular to river flow

$\therefore \ t = \frac{d}{v_b} = \frac{d}{\sqrt{v_{br}^2 - v_r^2}}$

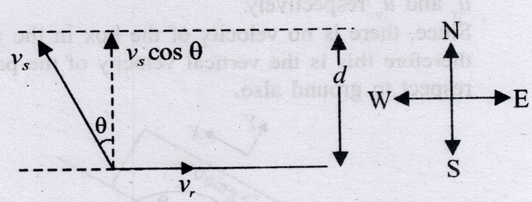


or $\ t = \frac{d}{v_b} = \frac{1\,\text{km}}{\frac{1}{4}}$

or $\ \frac{1}{4} = \frac{1}{\sqrt{25 - v_r^2}}$

$\Rightarrow v_r = 3$ km/h

2. **(a)** Time taken to cross the river $t = \frac{d}{v_s\cos\theta}$



For time to be minimum, $\cos\theta$ = maximum i.e., $\theta = 0°$

Hence swimmer should swim due north.